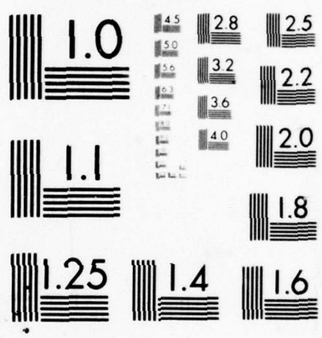
AD-A053 199 AIR FORCE GEOPHYSICS LAB HANSCOM AFB MASS F/G 4/2
PERSISTENCE, RUNS, AND RECURRENCE OF SKY COVER.(U)
DEC 77 I A LUND, D D GRANTHAM
UNCLASSIFIED AFGL-TR-77-0308 NL

1 OF 1
AD
A053199

END
DATE
FILMED
6-78
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A053199

AFGL-TR-77-0308
ENVIRONMENTAL RESEARCH PAPERS, NO. 621

(12)

SC

Persistence, Runs, and Recurrence of Sky Cover

IVER A. LUND
DONALD D. GRANTHAM

30 December 1977

METEOROLOGY DIVISION PROJECT 6670
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF

This report has been reviewed by the ESD Information Office (OI) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

John Howard

Chief Scientist

Qualified requestors may obtain additional copies from the Defense Documentation Center. All others should apply to the National Technical Information Service.

9 Environmental research papers,

14

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFGL-TR-77-0308, AFGL-ERP-621

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): 6 PERSISTENCE, RUNS, AND RECURRENCE OF SKY COVER.

5. TYPE OF REPORT & PERIOD COVERED: Scientific Interim

6. PERFORMING ORG. REPORT NUMBER: ERP No. 621

7. AUTHOR(s): 10
Iver A. Lund
Donald D. Grantham

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS:
Air Force Geophysics Laboratory (LYD)
Hanscom AFB,
Massachusetts 01731

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
62101F
16 66700904 17 Ø9

11. CONTROLLING OFFICE NAME AND ADDRESS:
Air Force Geophysics Laboratory (LYD)
Hanscom AFB,
Massachusetts 01731

12. REPORT DATE: 11 30 Dec [redacted] 77

13. NUMBER OF PAGES: 52

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report): Unclassified 12 53p.

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Sky cover
Persistence probabilities
Recurrence probabilities
Duration of cloudiness
Clouds

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A total of 511,056 hourly observations of total sky cover, taken over a thirteen-year period at nine stations, was studied to obtain a better understanding of the characteristics of persistence, runs, and recurrence. Each hourly total sky cover observation was categorized as either zero-tenths (clear), less than or equal to three-tenths, greater than or equal to eight-tenths, or ten-tenths (overcast). Probabilities of each category were estimated from relative frequencies determined from this large data sample and were compared with some theoretical models. The models can be applied to over

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

409 578 Gu

20. (Cont)

estimate probabilities that any of the above sky cover categories will be observed for sequences of x hours, or more; for exactly x hours; or at time t and also at time t+x hours.

Preface

The authors are grateful to James F. Atkinson, Leonard J. Natoli, Kenneth C. Zwirble, Analysis and Computer Systems, Inc., John F. Kellaher, Air Force Geophysics Laboratory, and Miss Melinda A. Zouvelos, Student Aid, Lower University, for expert computational support and to the USAF Environmental Technical Applications Center and the National Weather Service for hourly sky cover data.

Contents

1. INTRODUCTION 9

2. DATA 10

3. DATA PROCESSING 11

4. PERSISTENCE 11
   - 4.1 Observed 11
   - 4.2 Modeled 25

5. RUNS 30
   - 5.1 Observed 30
   - 5.2 Modeled 39

6. RECURRENCE 42
   - 6.1 Observed 42
   - 6.2 Modeled 49

7. REMARKS 51

REFERENCES 52

Illustrations

1. Location of the Nine Stations Whose Winter and Summer Hourly Observations of Total Sky Cover Were Studied 10

2. Relative Frequencies of Success, Given x Hours of Consecutive Successes Have Occurred, in Winter (a) and Summer (b) 17

Illustrations

3. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When Zero-tenths Sky Cover (clear) is Regarded as a Success 23

4. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When LE Three-tenths Sky Cover is Regarded as a Success 23

5. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When GE Eight-tenths Sky Cover is Regarded as a Success 24

6. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When Ten-tenths Sky Cover (overcast), is Regarded as a Success 24

7. Relative Frequencies of Success Given Exactly x Hours of Consecutive Success Have Occurred, in Winter (a) and Summer (b) 40

8. Relative Frequencies of a Success, Zero-tenths Sky Cover (clear) ℓ Hours Later Given a Success Has Occurred, in Winter (dots) and in Summer (X's) 42

9. Relative Frequencies of a Success, LE Three-tenths Sky Cover ℓ Hours Later Given a Success has Occurred in Winter (dots) and in Summer (X's) 43

10. Relative Frequencies of a Success, GE Eight-tenths Sky Cover, ℓ Hours Later Given a Success Has Occurred in Winter (dots) and in Summer (X's) 43

11. Relative Frequencies of a Success, Ten-tenths Sky Cover (overcast), ℓ Hours Later Given a Success Has Occurred, in Winter (dots) and in Summer (X's) 44

Tables

1. Sky Cover Categories 11

2. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a Success 13

3. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success 14

4. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From Data Sample When GE Eight-tenths Sky Cover is Considered a Success 15

5. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success 16

6. Median Values of $RF(S|S_x)$ Obtained From the Data Samples (Tables 2, 3, 4, and 5) and Probability Estimates $\hat{P}(S|S_x)$ Determined From Subjectively Drawn Curves of the Medians Shown in Figure 2 18

7. Relative Frequencies of x Consecutive Successes, $RF(S_x)$, Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a Success 19

8. Relative Frequencies of x Consecutive Successes, $RF(S_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success 20

9. Relative Frequencies of x Consecutive Successes, $RF(S_x)$, Obtained From the Data Sample When GE Eight-tenths Sky Cover is Considered a Success 21

10. Relative Frequencies of x Consecutive Successes $RF(S_x)$, Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success 22

11. The Minimum Number of Hours that Each Sky Cover Category was Estimated to Persist, at Selected Probability Levels (see text) 28

12. The Minimum Number of Hours that Each Sky Cover Category was Observed to Persist, Given that the Category is Observed, at Selected Probability Levels (see text) 29

13. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When Zero-tenths Sky Cover (clear) is Considered a Success 31

14. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When LE Three-tenths Sky Cover is Considered a Success 32

15. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When GE Eight-tenths Sky Cover is Considered a Success 33

16. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When Ten-tenths Sky Cover (overcast) is Considered a Success 34

17. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a Success 35

18. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success 36

19. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When GE Eight-tenths Sky Cover is Considered a Success 37

20. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success 38

Tables

21. Median Values of $RF(S|FS_x)$ Obtained From the Data Sample (Tables 17, 18, 19, and 20) and Probability Estimates $\hat{P}(S|FS_x)$ Determined From Subjectively Drawn Curves of the Medians Shown in Figure 7 41

22. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success Has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When Zero-tenths Sky Cover (clear) is Considered a Success 45

23. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success, Has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When LE Three-tenths Sky Cover is Considered a Success 46

24. Relative Frequency of the Recurrence of a Sucess ℓ Hours After a Success Has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When GE Eight-tenths Sky Cover is Considered a Success 47

25. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success Has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When Ten-tenths Sky Cover (overcast) is Considered a Success 48

26. Nine-station Median Relative Frequency of Each Sky Cover Category for Each Hour of the Day 50

27. The "a" Values Used in Eq. (17) to Find the Curves Shown in Figures 8 through 11 50

Persistence, Runs, and Recurrence of Sky Cover

1. INTRODUCTION

Duration, persistence, runs, and recurrence are all interrelated. For this study, they have been defined as follows: duration-continuous successes; persistence-consecutive successes separated by one hour; runs-consecutive successes separated by intervals of one hour beginning and ending with failure; and recurrence-successes occurring at time t and also at time t+x hours.

This study is part of a more comprehensive investigation conducted to obtain a better understanding of persistence, runs, and recurrence of weather events. Duration could not be studied because the data were observed at hourly intervals. Of major interest are those weather events which are usually recorded in categories, for example; precipitation recorded as none, light, moderate, or heavy; or sky cover recorded as clear, scattered, broken, or overcast. Persistence, runs, and recurrence of precipitation are described in a paper by Lund and Grantham.[1]

This report includes tables of observed relative frequencies of four sky-cover categories and models for estimating probabilities of each category. The models provide answers to such questions as: What is the probability of observing a sequence of more than five hours of overcast sky; of observing a run of exactly five hours of

(Received for publication 20 December 1977)

1. Lund, I. A., and Grantham, D. D. (1977) Persistence, runs, and recurrence of precipitation, J. Appl. Meteor. 16:346-358.

overcast; and, of observing overcast at time t and also at time t+5 hours? The models require a knowledge of the unconditional probability of the event, in this case a sky-cover category, and a measure of the temporal correlation between occurrences of sky-cover categories.

2. DATA

Records of hourly total sky cover observations taken in winter (December, January, February) and summer (June, July, August) during the 13-year period 1951 through 1963, at the following nine stations, shown in Figure 1, were studied:

LGA	LaGuardia Airport, New York, NY,
JFK	Kennedy International Airport, New York, NY,
EWR	Newark Airport, NJ,
PHL	Philadelphia International Airport, PA,
BAL	Baltimore-Washington International Airport, MD,
DCA	National Airport, Washington, DC,
ADW	Andrews AFB, MD,
RIC	Byrd Field, Richmond, VA,
RDU	Raleigh-Durham Airport, NC.

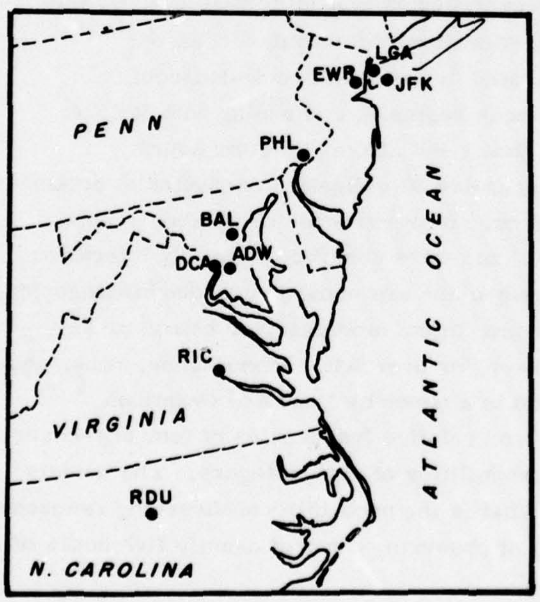


Figure 1. Location of the Nine Stations Whose Winter and Summer Hourly Observations of Total Sky Cover Were Studied

Each hour, approximately on the hour, a weather observer at each of the above stations went outdoors to make a regular hourly observation. One of the weather elements recorded is total sky cover. The Federal Meteorological Handbook[2] describes how the observations are taken. The four sky-cover categories studied are shown in Table 1. The abbreviations LE and GE stand for "less than or equal to" and "greater than or equal to", respectively. The two ends of the frequency distribution were studied separately to determine whether temporal correlation is a function of cloud amount.

Table 1. Sky Cover Categories

Category	Total Sky Cover
1	0.0 (clear)
2	LE 0.3 (0.0, 0.1, 0.2, and 0.3)
3	GE 0.8 (0.8, 0.9, and 1.0)
4	1.0 (overcast)

3. DATA PROCESSING

Each hourly sky cover observation was categorized as follows: Zero-tenths (clear), less than or equal to (LE) three-tenths, greater than or equal to (GE) eight-tenths and ten-tenths (overcast). Some of the stations had no missing observations, others only a very few. These few observations were filled in by estimating the sky cover from observations taken at nearby stations and observations taken before and after the missing observations. There were 28,080 [(24 observations/day) × (90 days/season) × (13 seasons)] observations, in winter and 28,704 [(24 observations/day) × (92 days/season) × (13 seasons)] observations, in summer, processed for each station.

4. PERSISTENCE

4.1 Observed

The occurrence of a given sky cover category was denoted as a success, S, and non-occurrence as a failure, F. The relative frequency of one success, $RF(S_1)$, is found from the data by dividing the number of times the sky cover category occurred,

2. U.S. Department of Commerce (1975) Federal Meteorological Handbook No. 1, Surface Observations, U.S. Government Printing Office, Washington, D.C. 309 pp.

$n(S_1)$ by the sample size N. The relative frequency of two successes in a row $RF(S_2)$, is found from the data by dividing the number of times a success was followed by a success, $n(S_2)$, by the sample size N minus the end effect, in this case 13, because there were 13 years when the next season's data were not used to determine the sky cover on the first hour of the next season. The relative frequency of x successes in a row, $RF(S_x)$, is found by dividing the number of times x consecutive successes was observed, $n(S_x)$, by the sample size, N, minus the end effects, in this case 13(x-1).

$$RF(S_x) = \frac{n(S_x)}{N-13(x-1)} \approx \frac{n(S_x)}{N} \qquad (1)$$

This processing of the data was done for all categories for all nine stations in both winter and summer.

The relative frequency of a success given that x consecutive successes have occurred, $RF(S|S_x)$, is equal to the relative frequency of x+1 consecutive successes, $RF(S_{x+1})$ divided by the relative frequency of x consecutive successes, $RF(S_x)$, that is,

$$RF(S|S_x) = \frac{RF(S_{x+1})}{RF(S_x)} \qquad (2)$$

The conditional relative frequencies $RF(S|S_x)$ were computed for periods up to 72 hours. Selected values for the first 15 hours are shown for all nine stations and all four sky cover categories in Tables 2, 3, 4, and 5. The median relative frequencies are indicated with asterisks.

The first column in each of the tables gives $RF(S|S_o)$ which is defined as $RF(S)$, the unconditional relative frequency of the given sky cover category. Although both the unconditional and conditional relative frequencies vary from station to station, there is often no consistent pattern to the variations. It was subjectively decided to assume that the data from all stations were drawn from the same sample and to use the median values to obtain estimates of the conditional probabilities $\hat{P}(S|S_x)$, required for obtaining estimates of joint probabilities, $\hat{P}(S_{x+1})$, that is, $\hat{P}(S_xS)$.

Table 2. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a Success. Median values are identified with asterisks

Season	Station	x (Hours)								
		0	1	3	5	7	9	11	13	15
Winter	LGA	.2481	.846	.869	.872	.871	.875	.871	.867	.868
	JFK	.2501	.847	.867	.871	.870	.870	.863	.864	.862
	EWR	.2634*	.860*	.878	.884	.881	.878	.875	.873	.875
	PHL	.2450	.840	.865	.870	.875	.876	.867	.870	.880
	BAL	.2737	.866	.889	.896	.897	.897	.895	.894*	.892
	ADW	.2540	.860*	.881*	.885*	.885*	.884*	.883*	.883	.883*
	DCA	.2716	.860*	.883	.893	.893	.896	.900	.901	.899
	RIC	.3073	.889	.901	.908	.908	.908	.906	.904	.903
	RDU	.3106	.886	.904	.914	.915	.915	.911	.913	.915
Summer	LGA	.1867	.782	.816	.822	.823	.837	.820	.835	.837
	JFK	.1833	.775	.803	.806	.806	.822	.837	.847	.839
	EWR	.2078*	.803	.826	.832	.835	.851	.852	.861	.867*
	PHL	.1920	.772	.804	.825*	.831*	.843*	.847*	.859*	.887
	BAL	.2301	.806	.840	.849	.851	.863	.858	.871	.890
	ADW	.1883	.797	.651	.819	.831*	.842	.839	.867	.883
	DCA	.2186	.793*	.826	.844	.847	.853	.862	.872	.888
	RIC	.2093	.806	.831	.841	.844	.852	.850	.854	.852
	RDU	.2101	.791	.820*	.824	.825	.818	.816	.824	.814

Table 3. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	x (Hours)								
		0	1	3	5	7	9	11	13	15
Winter	LGA	.3739	.882	.904	.910	.913	.912	.911	.911	.912
	JFK	.3821	.877	.900	.910	.914	.909	.909	.908	.909
	EWR	.3895	.885*	.907	.916	.917*	.917	.915	.915	.917
	PHL	.3636	.870	.899	.906	.908	.908	.908	.907	.910
	BAL	.3911	.887	.914	.920*	.922	.924	.922	.924*	.926*
	ADW	.3809	.889	.910*	.921	.906	.921*	.921*	.925	.926*
	DCA	.3849*	.884	.911	.920*	.925	.926	.927	.930	.929
	RIC	.4045	.897	.920	.926	.927	.928	.927	.927	.930
	RDU	.4194	.906	.925	.933	.937	.937	.936	.935	.934
Summer	LGA	.3703	.850	.880*	.889	.893	.894	.893	.892	.895
	JFK	.3719	.851*	.885	.892	.895*	.895	.895	.896	.903
	EWR	.3920	.859	.887	.896	.900	.901	.899*	.903	.903
	PHL	.3645	.835	.871	.885	.887	.896	.900	.907	.910
	BAL	.4270	.855	.886	.899	.907	.911	.915	.920	.926
	ADW	.3646	.853	.874	.887	.896	.900	.904	.902*	.906
	DCA	.4066	.852	.886	.897	.903	.909	.914	.916	.925
	RIC	.3729*	.842	.875	.891*	.894	.897*	.898	.900	.907*
	RDU	.3923	.842	.875	.884	.889	.890	.893	.894	.907*

Table 4. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From Data Sample When GE Eight-tenths Sky Cover Is Considered a Success. Median values are identified with asterisks

Season	Station	x (Hours) 0	1	3	5	7	9	11	13	15
Winter	LGA	.4962	.909*	.931	.940	.944	.946	.948	.947	.947
	JFK	.5022*	.906	.930	.940	.945	.947	.948	.948	.947
	EWR	.5073	.910	.934	.941*	.945	.949*	.951*	.949	.949
	PHL	.5306	.904	.933*	.943	.948	.951	.952	.952*	.953
	BAL	.4988	.908	.932	.939	.946*	.949*	.952	.952*	.937
	ADW	.5068	.912	.933*	.940	.945	.947	.951*	.951	.952*
	DCA	.5165	.910	.934	.942	.946*	.960	.951*	.952*	.953
	RIC	.5006	.913	.935	.943	.947	.950	.952	.954	.954
	RDU	.4761	.908	.933*	.941*	.946*	.948	.950	.953	.953
Summer	LGA	.4118	.865	.903	.914	.922	.930	.934*	.935	.936*
	JFK	.4376	.871	.903	.916	.925	.930	.935	.938	.940
	EWR	.4282	.871	.903	.918	.925	.931	.937	.940	.941
	PHL	.4387	.862*	.899	.912	.916	.924	.930	.933	.934
	BAL	.3738	.846	.889	.907	.918	.927*	.934*	.937	.938
	ADW	.4344	.865	.895*	.909*	.919*	.927*	.930	.936*	.936*
	DCA	.4033	.852	.890	.905	.916	.925	.934*	.938	.942
	RIC	.4160*	.850	.892	.908	.919*	.924	.930	.934	.935
	RDU	.3858	.844	.885	.894	.906	.918	.924	.928	.930

Table 5. Relative Frequencies of Success Given that x Consecutive Successes Have Occurred, $RF(S|S_x)$, Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success. Median values are identified with asterisks

Season	Station	x (Hours)								
		0	1	3	5	7	9	11	13	15
Winter	LGA	.4091	.892	.921	.933*	.939*	.941*	.940	.937	.937
	JFK	.4114	.894	.922	.933*	.939*	.940	.939	.940	.936
	EWR	.4262	.896*	.924*	.934	.940	.941*	.942*	.940	.941
	PHL	.4528	.897	.930	.940	.944	.946	.947	.947	.947
	BAL	.4155	.899	.924*	.934	.939*	.944	.945	.946	.945
	ADW	.4246	.897	.921	.931	.935	.940	.942*	.943	.943*
	DCA	.4395	.896*	.924*	.934	.939*	.943	.945	.946	.946
	RIC	.4175*	.899	.924*	.933*	.938	.940	.943	.945	.945
	RDU	.3855	.889	.920	.932	.937	.939	.942*	.944*	.942
Summer	LGA	.2801	.836	.887	.907	.916	.923	.926*	.930	.891
	JFK	.3015	.837	.880	.899	.908*	.920*	.923	.928*	.927
	EWR	.3050	.846	.890	.902	.888	.919	.927	.964	.929
	PHL	.3188	.824*	.877	.894	.908*	.943	.921	.927	.883
	BAL	.2526	.825	.873*	.892*	.909	.923	.971	.930	.934
	ADW	.2889	.817	.862	.883	.902	.916	.926*	.930	.932
	DCA	.2824*	.812	.866	.892*	.912	.921	.926*	.927	.928*
	RIC	.2736	.815	.864	.865	.928	.907	.916	.921	.929
	RDU	.2534	.796	.845	.859	.880	.899	.910	.922	.924

The median values of RF($S|S_x$) for winter and summer are shown in Figure 2 for periods up to 15 hours. The median relative frequency, RF(S), of the most frequently occurring category, GE 0.8, was 0.5022 in winter and 0.4160 in summer. Because this is a frequently occurring category there were many long sequences of successes. The median conditional relative frequencies for GE 0.8, given in Table 6 and shown as x's in Figure 2, increase in magnitude for 13 hours in winter and 12 hours in summer. They never vary significantly, that is, by more than 0.015, for the next few hours after hour 15, therefore the estimated conditional probabilities are regarded as constant after hour 15. The conditional relative frequencies of the less frequently occurring categories are more variable as expected but they never depart from the hour 15 conditional relative frequencies by more than 0.069.

Sample relative frequencies of x consecutive successes, RF(S_x), obtained from the data sample, are given in Tables 7, 8, 9, and 10. All of the relative frequencies for hours 1 through 70 are shown in Figures 3, 4, 5, and 6.

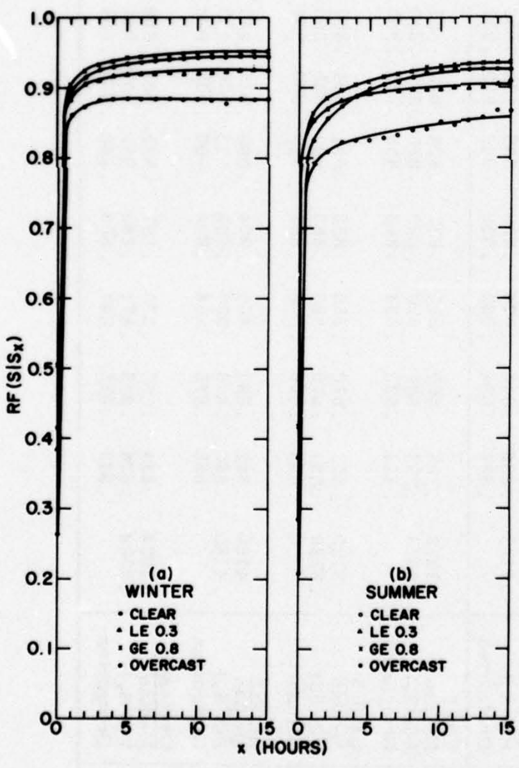


Figure 2. Relative Frequencies of Success, Given x Hours of Consecutive Successes Have Occurred, in Winter (a) and Summer (b). The curves were subjectively drawn

Table 6. Median Values of RF($S|S_x$) Obtained From the Data Samples (Tables 2, 3, 4, and5) and Probability Estimates $\hat{P}(S|S_x)$ Determined From Subjectively Drawn Curves of the Medians Shown in Figure 2. Also shown are some probability estimates obtained from Gringorten's model

Season		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Winter	Clear																
	Median	.2634	.860	.872	.881	.883	.885	.884	.885	.884	.884	.884	.883	.879	.883	.884	.883
	$\hat{P}(S\|S_x)$	.2634	.860	.871	.880	.882	.884	.884	.884	.884	.884	.884	.884	.884	.884	.884	.884
	Gringorten		.840	.863	.875	.884	.892	.900	.906	.911	.917	.920	.922	.923	.925	.926	.927
	LE 0.3																
	Median	.3849	.885	.903	.910	.916	.920	.921	.917	.921	.921	.920	.921	.923	.924	.925	.926
	$\hat{P}(S\|S_x)$	.3849	.885	.903	.912	.918	.920	.920	.920	.920	.920	.921	.922	.923	.924	.925	.925
	Gringorten		.889	.897	.904	.910	.915	.920	.924	.928	.932	.934	.936	.938	.939	.941	.942
	GE 0.8																
	Median	.5022	.909	.925	.933	.938	.941	.943	.946	.947	.949	.950	.951	.951	.952	.952	.952
	$\hat{P}(S\|S_x)$	.5022	.909	.925	.933	.938	.941	.943	.946	.947	.949	.950	.951	.952	.952	.952	.952
	Gringorten		.910	.918	.926	.932	.936	.940	.944	.946	.948	.949	.950	.951	.951	.952	.952
	Overcast																
	Median	.4175	.896	.914	.924	.930	.933	.937	.939	.940	.941	.942	.942	.943	.944	.943	.943
	$\hat{P}(S\|S_x)$	.4175	.896	.914	.924	.930	.935	.938	.939	.940	.941	.942	.942	.943	.943	.943	.943
	Gringorten		.884	.894	.902	.910	.917	.922	.927	.932	.936	.938	.941	.942	.943	.943	.943
Summer	Clear																
	Median	.2078	.793	.809	.820	.823	.825	.829	.831	.838	.843	.851	.847	.852	.859	.868	.867
	$\hat{P}(S\|S_x)$	.2078	.793	.809	.820	.829	.835	.839	.842	.848	.849	.850	.851	.953	.859	.859	.859
	Gringorten		.814	.828	.838	.848	.855	.862	.868	.875	.880	.885	.889	.893	.895	.897	.898
	LE 0.3																
	Median	.3729	.851	.870	.880	.886	.891	.894	.895	.897	.897	.898	.899	.899	.902	.904	.907
	$\hat{P}(S\|S_x)$	.3729	.851	.870	.880	.886	.892	.895	.899	.900	.902	.902	.903	.903	.904	.904	.904
	Gringorten		.855	.866	.876	.885	.893	.900	.906	.912	.926	.928	.930	.932	.933	.933	.933
	GE 0.8																
	Median	.4160	.862	.883	.895	.904	.909	.915	.919	.923	.927	.930	.934	.933	.936	.936	.936
	$\hat{P}(S\|S_x)$	.4160	.862	.883	.895	.905	.913	.920	.923	.927	.929	.930	.931	.933	.936	.936	.936
	Gringorten		.863	.875	.886	.894	.900	.905	.908	.913	.915	.917	.919	.920	.921	.922	.922
	Overcast																
	Median	.2824	.824	.855	.873	.887	.892	.902	.908	.914	.920	.922	.926	.924	.928	.928	.928
	$\hat{P}(S\|S_x)$	.2824	.824	.855	.871	.885	.897	.903	.912	.916	.921	.923	.926	.927	.928	.929	.929
	Gringorten		.815	.835	.847	.858	.868	.877	.884	.892	.897	.902	.906	.910	.914	.917	.920

Table 7. Relative Frequencies of x Consecutive Successes, RF(S_x), Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a success. Median values are identified with asterisks

Season	Station	1	2	3	4	5	6	9	12	18	24	30	36	48	60	72
Winter	LGA	.248	.210	.181	.157	.137	.120	.0790	.0526	.0228	.0105	.00517	.00178	.000178	0	0
	JFK	.250	.212	.182	.158	.138	.120	.0790	.0514	.0215	.00973	.00503	.00192	0	0	0
	EWR	.263*	.226*	.197*	.173*	.153*	.135*	.0924*	.0623	.0278	.0129	.00667	.00278	.000143	0	0
	PHL	.245	.206	.177	.153	.133	.115	.0771	.0511	.0229	.0115	.00570	.00232	.000606	.000178*	0
	BAL	.274	.237	.208	.185	.165	.148	.107	.0768	.0389	.0219	.0130	.00759	.00246	.000999	.000143
	ADW	.254	.219	.191	.168	.148	.131	.0908	.0627*	.0297*	.0155*	.00841*	.00435*	.00132*	.000892	.000464
	DCA	.272	.234	.205	.181	.161	.144	.103	.0743	.0392	.0219	.0124	.00610	.00177	.00103	.000607
	RIC	.307	.273	.245	.220	.200	.181	.136	.101	.0555	.0321	.0177	.00763	.00136	.0000357	0
	RDU	.311	.275	.247	.224	.203	.186	.142	.108	.0636	.0386	.0228	.0127	.00417	.000964	.000143
	Median	.263	.226	.197	.173	.153	.135	.0924	.0627	.0297	.0155	.00841	.00435	.00132	.000178	0
Summer	LGA	.187	.146	.117	.0958	.0787	.0647	.0361	.0207	.00722	.00293	.00101	.000453	0	0	0
	JFK	.183	.142	.113	.0907	.0730	.0589	.0310	.0178	.00648	.00258	.00136	.000767	0	0	0
	EWR	.208*	.167	.137	.113	.0935	.0779	.0455	.0281	.0120	.00662	.00394*	.00237*	.00143*	.00101	.000594
	PHL	.192	.148	.118	.0948	.0772	.0638	.0367	.0223	.0100*	.00624	.00439	.00328	.00202	.00119	.000454
	BAL	.230	.186	.153	.128	.109	.0925	.0570	.0365	.0174	.0107	.00757	.00527	.00286	.00161	.00119
	ADW	.188	.150	.121	.0976	.0797	.0653	.0377	.0223	.0100*	.00596*	.00453	.00366	.00234	.00112	.000664
	DCA	.219	.173	.141	.116	.0976	.0823	.0499	.0315	.0150	.00934	.00628	.00394	.00150	.000279	0
	RIC	.209	.169	.139	.115	.0962	.0808	.0486	.0301	.0117	.00537	.00300	.00216	.00126	.000698*	.000279*
	RDU	.210	.166*	.134*	.110*	.0906*	.0747*	.0414*	.0227*	.00687	.00310	.00181	.000802	.000105	0	0
	Median	.208	.166	.134	.110	.0906	.0747	.0414	.0227	.0100	.00596	.00394	.00237	.00143	.000698	.000279

Table 8. Relative Frequencies of x Consecutive Successes, $RF(S_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	1	2	3	4	5	6	9	12	18	24	30	36	48	60	72
Winter	LGA	.374	.330	.296	.267	.243	.221	.168	.127	.0726	.0395	.0223	.0128	.00428	.000892	0
	JFK	.382	.335	.299	.269	.245	.223	.169	.127	.0715	.0405	.0237	.0138	.00482	.00182	.000964
	EWR	.389	.345	.310	.281	.257	.235*	.182	.140	.0827	.0496	.0309	.0199	.00706	.00243	.000893
	PHL	.364	.316	.282	.254	.229	.207	.155	.116	.0658	.0392	.0243	.0148	.0539	.00157	.000464
	BAL	.391	.347	.314	.287	.264	.243	.191	.150	.0944	.0619	.0417	.0288	.0133	.00567	.00278
	ADW	.381	.338	.307*	.279	.256*	.235*	.184*	.144*	.0904*	.0601*	.0410*	.0271*	.0104*	.00492	.00236
	DCA	.385*	.340*	.307*	.280*	.256*	.236	.187	.149	.0958	.0650	.0448	.0300	.0119	.0528	.00218
	RIC	.404	.363	.331	.304	.281	.260	.207	.165	.106	.0703	.0459	.0288	.0107	.00371*	.00128*
	RDU	.419	.380	.349	.323	.300	.280	.230	.189	.126	.0834	.0541	.0346	.0171	.0842	.00339
Median		.385	.340	.307	.280	.256	.235	.184	.144	.0904	.0601	.0410	.0271	.0104	.00371	.00128
Summer	LGA	.370	.315	.274	.241	.213	.190	.136*	.0967	.0498	.0285	.0174	.00949	.00331	.00147	.000733
	JFK	.372	.317*	.277*	.245*	.218*	.195	.139	.0991	.0532*	.0320	.0205	.0122	.00555	.00325*	.00157
	EWR	.392	.337	.295	.262	.234	.210	.153	.111	.0604	.0352	.0220*	.0132*	.00541	.00325*	.00206
	PHL	.364	.304	.261	.228	.200	.177	.124	.0898	.0512	.0322	.0206	.0129	.00579*	.00356	.00189*
	BAL	.427	.365	.320	.283	.253	.228	.170	.130	.0801	.0518	.0347	.0226	.0103	.00457	.00234
	ADW	.365	.311	.270	.236	.208	.184	.133	.0971*	.0537	.0338*	.0239	.0151	.00726	.00370	.00192
	DCA	.407	.346	.303	.268	.239	.215	.158	.120	.0736	.0500	.0350	.0244	.0114	.00614	.00374
	RIC	.373*	.314	.271	.237	.210	.187	.134	.0971	.0525	.0306	.0190	.0120	.00488	.00251	.00161
	RDU	.392	.330	.285	.249	.220	.194*	.136*	.0965	.0528	.0350	.0248	.0176	.00754	.00237	.000733
Median		.373	.317	.277	.245	.218	.194	.136	.0971	.0532	.0338	.0220	.0132	.00579	.00325	.00189

Table 9. Relative Frequencies of x Consecutive Successes, RF(S_x), Obtained From the Data Sample When GE Eight-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	1	2	3	4	5	6	9	12	18	24	30	36	48	60	72
Winter	LGA	.496	.451	.416	.387	.362	.340	.286	.243	.175	.125	.0887	.0629	.0355	.0206	.0118
	JFK	.502*	.455	.420	.390	.365	.343	.289	.246	.177	.126	.0883	.0618	.0332	.0188	.0106
	EWR	.507	.462	.427	.399	.374	.352	.298	.255	.186*	.133	.0942	.0669	.0367	.0211	.0126
	PHL	.531	.479	.442	.412	.387	.365	.310	.267	.199	.148	.110	.0827	.0490	.0298	.0170
	BAL	.499	.453	.419	.390	.365	.343	.290	.249	.185	.138	.104*	.0794*	.0483	.0299	.0178
	ADW	.507	.462	.428	.399	.374	.351*	.296*	.253*	.188	.141	.105	.0796	.0465*	.0263*	.0146*
	DCA	.516	.470	.436	.407	.382	.360	.304	.260	.194	.145	.110	.0835	.0497	.0297	.0167
	RIC	.501	.457*	.425*	.397*	.373*	.351*	.299	.257	.194	.146	.110	.0829	.0499	.0294	.0167
	RDU	.476	.432	.400	.373	.350	.329	.278	.238	.178	.133	.0991	.0734	.0410	.0221	.0116
	Median	.502	.457	.425	.397	.373	.351	.296	.253	.186	.138	.104	.0794	.0465	.0263	.0146
Summer	LGA	.412	.356*	.317*	.286*	.261*	.238*	.187*	.152	.102	.0706	.0498	.0345	.0152*	.00583	.00126
	JFK	.438	.381	.340	.307	.280	.257	.203	.164	.113	.0765	.0522	.0348	.0152*	.00653*	.00293
	EWR	.428	.373	.332	.300	.273	.251	.199	.162	.112	.0778	.0541	.0368	.0172	.00695	.00272*
	PHL	.439	.378	.334	.301	.272	.248	.191	.152	.101*	.0698*	.0489	.0339	.0147	.00478	.000908
	BAL	.374	.316	.277	.246	.221	.201	.155	.125	.0848	.0599	.0431	.0299	.0142	.00670	.00311
	ADW	.434	.376	.332	.297	.269	.244	.189	.151*	.102	.0700	.0483*	.0323*	.0162	.00845	.00381
	DCA	.403	.343	.300	.267	.240	.217	.167	.134	.0924	.0642	.0447	.0311	.0165	.00855	.00388
	RIC	.416*	.354	.310	.277	.249	.226	.175	.140	.0932	.0629	.0425	.0284	.0136	.00649	.00237
	RDU	.386	.326	.284	.251	.224	.200	.149	.116	.0750	.0500	.0334	.0226	.00977	.00332	.000873
	Median	.416	.356	.317	.286	.261	.238	.187	.151	.101	.0698	.0483	.0323	.0152	.00653	.00272

Table 10. Relative Frequencies of x Consecutive Successes RF(S_x), Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success. Median values are identified with asterisks

Season	Station	1	2	3	4	5	6	9	12	18	24	30	36	48	60	72
Winter	LGA	.409	.365	.332	.306	.284	.265	.219	.182	.123	.0821	.0559	.0395	.0227	.0137	.00810
	JFK	.411	.368	.335	.309	.288	.269	.222	.184	.125	.0833	.0559	.0385	.0206	.0126	.00746
	EWR	.426	.382	.349	.323	.300	.280	.232	.194	.134*	.0915	.0641	.0469	.0269*	.0167	.0100
	PHL	.453	.406	.373	.347	.325	.305	.257	.218	.157	.112	.0815	.0600	.0340	.0196	.0102
	BAL	.415	.374	.342	.316	.294	.275*	.228	.192	.137	.0984	.0718	.0536	.0300	.0175	.0103
	ADW	.425	.381	.348	.320	.297	.276	.227*	.189	.132	.0932*	.0659*	.0472*	.0241	.0131	.00710
	DCH	.439	.394	.360	.333	.309	.289	.239	.202	.144	.103	.0751	.0554	.0302	.0172	.00939*
	RIC	.417*	.376*	.343*	.317*	.295*	.275*	.227*	.190*	.135	.0959	.0690	.0509	.0300	.0165*	.00950
	ADU	.385	.343	.312	.287	.266	.248	.204	.169	.120	.0847	.0586	.0400	.0188	.00846	.00325
	Median	.417	.376	.343	.317	.295	.274	.227	.190	.134	.0932	.0659	.0472	.0269	.0165	.00939
Summer	LGA	.280	.234*	.203	.180	.162	.147	.113	.0895	.0586	.0390	.0258	.0163	.00628	.00241*	.000279
	JFK	.301	.252	.218	.192	.171	.154	.115	.0901	.0573	.0362	.0232	.0149	.00548	.00209	.000524
	EWR	.305	.258	.226	.201	.180	.162	.123	.0964	.0614	.0396	.0256	.0158	.00485	.00129	0
	PHL	.319	.263	.225	.197	.175	.156	.116	.0899	.0571	.0375	.0247	.0165	.00618*	.00223	.000629*
	BAL	.253	.208	.178	.156	.137	.122	.0919	.0727	.0476	.0316	.0212	.0139*	.00666	.00265	.000908
	ADW	.289	.236	.200*	.172*	.150*	.133*	.0975	.0764	.0495	.0322*	.0213*	.0139*	.00649	.00328	.00119
	DCA	.282*	.229	.194	.168	.148	.132	.100*	.0788*	.0502*	.0321	.0208	.0131	.00649	.00311	.00115
	RIC	.274	.223	.188	.163	.143	.126	.0918	.0695	.0436	.0284	.0186	.0117	.00555	.00276	.00129
	ADU	.253	.202	.167	.142	.121	.104	.0702	.0520	.0320	.0205	.0131	.0841	.00342	.000873	0
	Median	.282	.234	.200	.172	.150	.133	.100	.0788	.0502	.0322	.0213	.0139	.00618	.00241	.000629

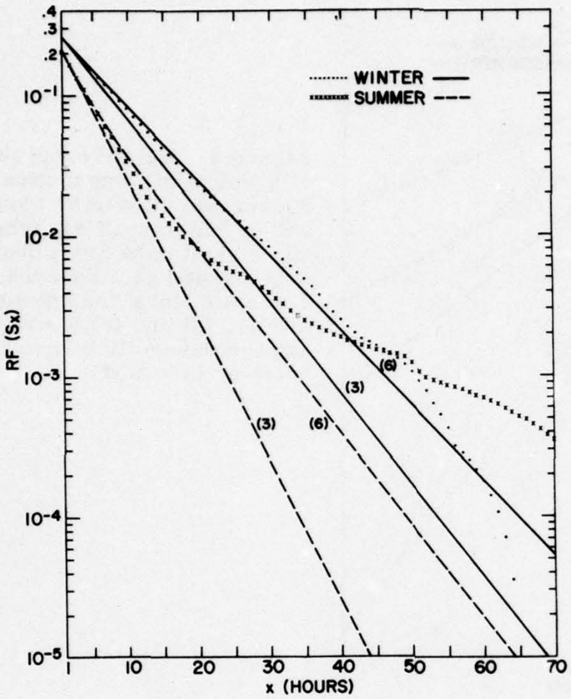


Figure 3. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), Where Zero-tenths Sky Cover (clear) is Regarded as a Success. The solid lines are solutions to Eqs. (3) and (6) for Winter and the dashed lines are for Summer (see text)

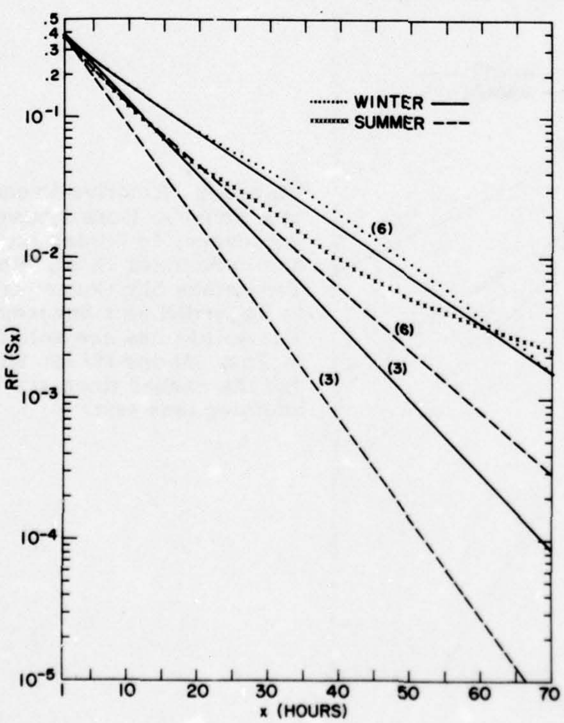


Figure 4. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When LE Three-tenths Sky Cover is Regarded as a Success. The solid lines are solutions to Eqs. (3) and (6) for Winter and the dashed lines are for Summer (see text)

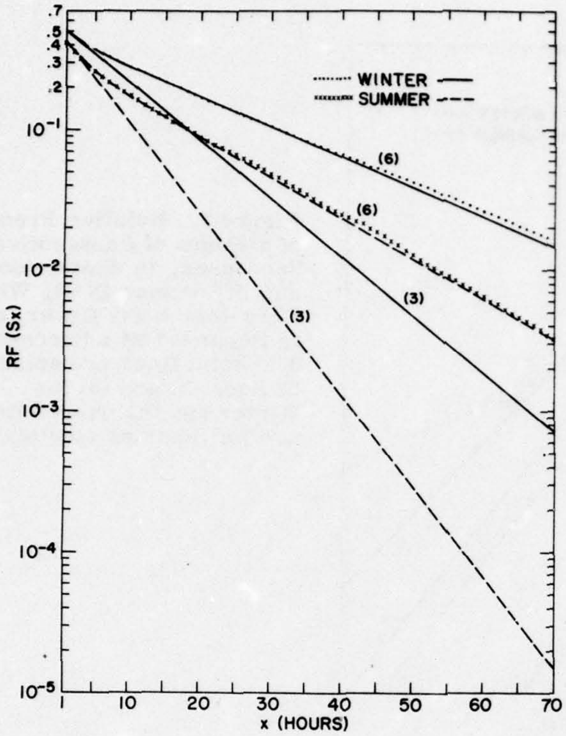


Figure 5. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When GE Eight-tenths Sky Cover is Regarded as a Success. The solid lines are solutions to Eqs. (3) and (6) for Winter and the dashed lines are for Summer (see text)

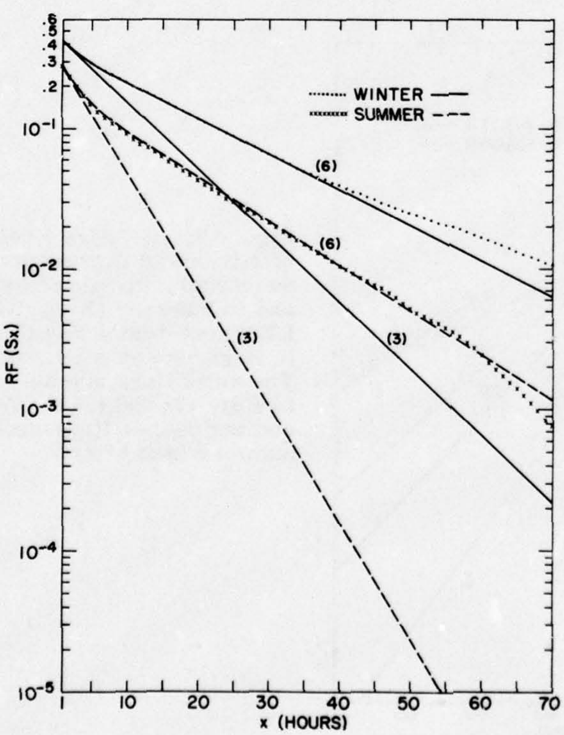


Figure 6. Relative Frequencies of x Hours of Consecutive Successes, in Winter (dots) and in Summer (X's), When Ten-tenths Sky Cover (overcast), is Regarded as a Success. The solid lines are solutions to Eqs. (3) and (6) for Winter and the dashed lines are for Summer (see text)

4.2 Modeled

The probability of a sequence of x hours of successes is sometimes estimated with the following first order Markov chain

$$\hat{P}(S_x) = P(S)\,[\,P(S|S_1)]^{x-1} \qquad (3)$$

where $P(S|S_1)$ is the probability of a success given a success has occurred and x equals the number of hours.

The relative frequencies RF(S) and $RF(S|S_1)$, obtained from the data, are given in the first two columns in Tables 2 through 5. The estimated conditional probabilities, $\hat{P}(S|S_1)$, shown in Table 6, were used to test Eq. (3). The model fit the observed values, within a few percent, for the first few hours but there were large differences between the model estimates and corresponding sample relative frequencies when probabilities of sequences of successes longer than a few hours were estimated. Figures 3 to 6 illustrate differences between model estimates and observed relative frequencies when the nine station median relative frequencies are used to represent the sample values. These figures illustrate the failure of Eq. (3) to adequately estimate long sequences of successes. For example, note the large departures of the first order Markov model estimates, labelled (3), from the sample relative frequencies of overcast sky cover shown in Figure 6.

To improve the model given in Eq. (3) the following axiomatic expressions can be used to estimate two and x hours of consecutive successes, respectively

$$P'(S_2) = P(S)\,P(S|S_1) \qquad (4)$$

$$P'(S_x) = P(S)\,P(S|S_1)\ldots P(S|S_{x-1}) \qquad x \geq 3 \qquad (5)$$

where $P(S|S_1)$ is the probability of a success given that a success occurred the previous hour and $P(S|S_{x-1})$ is the probability of a success following (x-1) hours of unbroken successes.

The probabilities required for the solution of Eq. (5) were estimated from the relative frequencies and it was assumed that the conditional probabilities were always constant beyond 15 hours. To estimate joint probabilities Eq. (5) was expressed as follows

$$\left.\begin{array}{ll} \hat{P}'(S_x) = \hat{P}(S)\,\hat{P}(S|S_{x-1}) & x = 2 \\ \hat{P}'(S_x) = \hat{P}(S)\,\hat{P}(S|S_1)\ldots\ldots\hat{P}(S|S_{x-1}) & 3 \leq x \leq 15 \\ \hat{P}'(S_x) = \hat{P}(S)\,\hat{P}(S|S_1)\ldots\ldots\hat{P}(S|S_{14})\,[\,\hat{P}(S|S_{15})]^{x-15} & x \geq 16 \end{array}\right\} \qquad (6)$$

Curves were drawn for the points in Figure 2 and estimates of $\hat{P}(S|S_x)$ for use in Eq. (6) were obtained from the curves. These values are given in Table 6 in the rows labelled $\hat{P}(S|S_x)$. The winter curves in Figure 2 follow the median values very closely but the summer curves were drawn smoothly without following as near to the data points. The departures of the points representing the clear and LE three-tenths categories from the subjectively drawn curves are not understood, but they may be related to the diurnal period in these two sky cover categories.

Solutions to Eq. (6) are shown by the curves in Figures 3 to 6. They are in better agreement with the relative frequencies than curves based on Eq. (3). This must be the case, because the probability estimates for Eq. (6) are based more closely on the relative frequencies. Modeling is involved in the smoothing of the relative frequencies and in the assumption that $P(S|S_x)$ is constant beyond 15 hours.

Gringorten[3] simulated probability distributions by a Monte Carlo exercise and prepared charts for use in estimating the duration of weather events. These charts were used to estimate conditional probabilities of the four categories of total sky cover. Hour-to-hour correlation for this application of Gringorten's method was assumed to be 0.950 in winter and 0.932 in summer. Table 6 shows that the conditional probability estimates obtained by Gringorten's method are in generally good agreement with the observed values. One shortcoming of the method is that it is graphical. It is difficult to estimate the probabilities from the charts. A promising analytical method described in the treatise by Keilson and Ross[4] needs further development before it can be applied to this problem. In the absence of conditional relative frequencies obtained from large data samples, Gringorten's method can provide suitable estimates of the probabilities required for the solution of Eq. (6). Because a large sample of data was available for this study, a smooth subjective fit to the relative frequencies was used to estimate the required probabilities.

Table 11 summarizes some of the information obtainable from Eq. (6) and the data points shown in Figures 3 to 6. It shows the number of hours, x, that each of the four sky cover categories was estimated, and observed, to persist, at six probability levels based on $\hat{P}(S_x)$ and $RF(S_x)$, respectively. For example, Table 11 shows that clear (0.0 sky cover) has a climatic occurrence probability of 0.2634, in winter. Therefore, more than 50% of the time clear would not be expected and a zero is entered in the table. However, more than 25% of the time clear is expected to be observed for at least one hour. Clear is expected to persist for 8, 14, 27, and 46 hours; 10%, 5%, 1%, and 0.1% of the time respectively, as can be seen from the solid curve based on Eq. (6) shown in Figure 3. The corresponding observed values are shown in parentheses in the table.

3. Gringorten, I. I. (1966) A stochastic model of the frequency and duration of weather events, J. Appl. Meteor. 5:606-624.

4. Keilson, J., and Ross, H. F. (1975) Passage time distributions for Gaussian Markov (Ornstein-Uhlenbeck) statistical processes, Selected Tables in Mathematical Statistics Vol. III, American Mathematical Society, Providence, Rhode Island, pp 233-327.

The values given in Tables 11 and 12 are for east-coast stations. They apply elsewhere only to the extent that the probability of the event and the hour-to-hour correlation is the same.

Table 12 summarizes some of the information obtainable from the following equation,

$$\hat{P}(S_x | S) = \frac{P(S_{x+1})}{P(S)} . \tag{7}$$

This equation is used to estimate the conditional probability of observing a sequence of x hours of a weather category. It can be used to answer questions such as: given that it is clear, how many hours will it be before there is a 50% probability that the sequence of clear skies will be broken. The unconditional probability, $P(S)$, is assumed to be known and $\hat{P}(S_x | S)$ must equal 0.50. Substituting, for example, the winter unconditional probability of observing clear, 0.2634, into Eq. (7), it becomes

$$0.50 = \frac{P(S_{x+1})}{0.2634} \tag{7a}$$

$$P(S_{x+1}) = 0.1317 . \tag{7b}$$

Solutions to Eq. (6) for $\hat{P}(S_{x+1})$ for clear are shown in Figure 3. It can be seen that $\hat{P}(S_{x+1}) = 0.1317$ when $x+1 \approx 6$ hours, therefore, clear is expected to persist for more than 5 hours about 50% of the time. The dots in Figure 3 show values of $RF(S_x)$, for winter. It can be seen that $RF(S_x) = 0.1317$ when $x=6$ hours. At this point the model and the data are in good agreement.

Table 12 shows that from Eq. (7), when clear is observed, it is expected to persist 5, 11, 19, 24, 37, and 56 hours; 50%, 25%, 10%, 5%, 1%, and 0.1% of the time, respectively. Corresponding observed values are shown in parentheses.

The following equation can be used to estimate how long a sequence of successes is expected to persist, given that the sequence has just begun,

$$\hat{P}(S_x | FS) = \frac{P(FS_{x+1})}{P(FS)} \tag{8}$$

The F preceeding the S denotes a failure followed by a success.

Eq. (8) always yields smaller values than Eq. (7), unless the process is first-order Markov in which case the values are identical. A table of values based on solutions to Eq. (8) was not prepared, but sufficient information is included in this report to prepare such a table.

Table 11. The Minimum Number of Hours That Each Sky Cover Category Was Estimated to Persist, at Selected Probability Levels (see text). Observed values are shown in parenthesis

Category	Season	Climatic Probability	Probability 50%	25	10	5	1	0.1
Clear (0.0)	W	.2634	0 (0)	1 (1)	8 (8)	14 (14)	27 (28)	46 (50)
	S	.2078	0 (0)	0 (0)	4 (4)	8 (8)	19 (18)	34 (52)
LE 0.3	W	.3849	0 (0)	5 (5)	16 (16)	25 (27)	46 (48)	>72 (>72)
	S	.3729	0 (0)	4 (4)	12 (12)	19 (19)	35 (39)	57 (>72)
GE 0.8	W	.5022	1 (1)	12 (12)	31 (31)	45 (46)	>72 (>72)	
	S	.4160	0 (0)	5 (5)	18 (18)	28 (29)	53 (53)	>72 (>72)
Overcast (1.0)	W	.4175	0 (0)	7 (7)	23 (23)	35 (35)	62 (71)	>72 (>72)
	S	.2824	0 (0)	1 (1)	9 (9)	19 (18)	41 (41)	72 (68)

Table 12. The Minimum Number of Hours That Each Sky Cover Category Was Observed to Persist, Given That the Category is Observed, at Selected Probability Levels (see text). Observed values are shown in parenthesis

Category	Season	Climatic Probability	Probability					
			50%	25	10	5	1	0.1
Clear (0.0)	W	.2634	5 (5)	11 (11)	19 (18)	24 (25)	37 (39)	56 (57)
	S	.2078	3 (3)	7 (7)	13 (12)	18 (17)	28 (37)	43 (>72)
LE 0.3	W	.3849	7 (7)	16 (16)	28 (30)	37 (39)	57 (58)	>72 (>72)
	S	.3729	6 (6)	12 (12)	21 (22)	28 (31)	44 (56)	>72 (>72)
GE 0.8	W	.5022	11 (11)	25 (25)	44 (45)	58 (60)	>72 (>72)	>72 (>72)
	S	.4160	7 (7)	16 (16)	30 (31)	40 (42)	65 (66)	>72 (>72)
Overcast (1.0)	W	.4175	9 (9)	21 (21)	37 (37)	49 (53)	>72 (>72)	>72 (>72)
	S	.2824	5 (5)	13 (13)	25 (25)	35 (35)	57 (58)	>72 (>72)

5. RUNS

5.1 Observed

Another way of examining persistence is to consider the number of runs of exactly x hours in length, that is, $n(FS_1F)$, $n(FS_2F)\ldots$, $n(FS_xF)$. The relative frequency of runs is given by the expression

$$RF(FS_xF) = \frac{n(FS_xF)}{N-13(x-1)} \approx \frac{n(FS_xF)}{N} \tag{9}$$

where $n(FS_xF)$ is the observed number of runs of exactly x hours in length and N is the total number of hours in the data sample.

The observed number of runs, based on 28,080 hours of winter observations, and 28,704 hours of summer observations, at each of the nine stations is given, for selected hours, in Tables 13 to 16. The median values are indicated with asterisks. Although the frequencies are based on more than 28,000 observations at each station and season, there are large sampling variations. For example, in Table 13, in winter, ADW had only 88 runs of three hours of clear while the nearby station of DCA had 122, LGA had only 4 runs of eighteen hours while the nearby station JFK had 14, and RDU had 3 runs of 24 hours but 8 runs of 36 hours.

To model the runs it is assumed that one good model can estimate runs at any of the nine stations, at least as well as a 13-year data sample.

A model was considered that is very similar to Eq. (6). This model requires estimates of the conditional probabilities, $\hat{P}(S|FS_x)$. Relative frequencies of success given a failure and x hours of successes were determined from the data with the following expression

$$RF(S|FS_x) = \frac{n(FS_{x+1})}{n(FS_x)} . \tag{10}$$

These relative frequencies, a selection of which are given in Tables 17 to 20, were used to obtain the required conditional probabilities.

Table 13. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When Zero-tenths Sky Cover (clear) is Considered a Success. Median values are identified with asterisks

Season	Station	x (hours) 1	2	3	4	5	6	12	18	24	30	36
Winter	LGA	251	154	108	66	55	59	26	4	1	1	1
	JFK	244	146	115	68	64	55	29*	14	3*	3	1
	EWR	220	140	103*	75	57	40	33	11	8	1	3*
	PHL	294	137	104	84	70	59	32	10	2	2*	4
	BAL	233*	149	90	80	54*	42	30	17	2	2*	3*
	ADW	213	144	88	73*	47	58	32	13	4	3	0
	DCA	253	142*	122	64	53	46	24	12*	4	5	1
	RIC	157	117	98	69	45	48*	25	16	5	3	8
	RDU	202	122	94	79	41	46	24	6	3*	1	8
	Median	233	142	103	73	54	48	29	12	3	2	3
	$\hat{n}(FS_xF)$	236	147	100	74	56	47	23	12	6	3	2
Summer	LGA	345	201*	131	88	61	72	21*	4	1*	1	0*
	JFK	343	201*	132	100	83	58	14	7	3	0*	0*
	EWR	307	187	129	103*	81	61	19	6	0	0*	0*
	PHL	389	204	159	118	69	63	17	1	0	0*	1
	BAL	345*	231	144	85	75*	62	23	9	1*	0*	0*
	ADW	254	186	144	102	93	57	26	7*	1*	1	0*
	DCA	367	226	162	104	73	59	20	4	0	2	0*
	RIC	309	183	126	109	75*	60*	21*	10	1*	0*	0*
	RDU	349	219	133*	104	77	71	22	11	0	0*	0*
	Median	345	201	133	103	75	60	21	7	1	0	0
	$\hat{n}(FS_xF)$	324	204	134	98	73	57	19	7	2	1	0

Table 14. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When LE Three-tenths Sky Cover is Considered a Success. Median values are identified with asterisks.

Season	Station	x (hours) 1	2	3	4	5	6	12	18	24	30	36
Winter	LGA	283	158	117	72	61	58	30	20	16	5*	2
	JFK	313	165	142	79*	68	57	32	11	8	6	4
	EWR	286	165	119	83	50	54	26*	14	8	5*	2
	PHL	352	174	111*	79*	64	58	24	6	7*	2	3*
	BAL	321	164	105	58	57*	48*	29	20	6	5*	3*
	ADW	300*	120	113	87	53	35	28	17*	4	4	3*
	DCA	324	163*	111*	84	71	45	22	16	6	3	2
	RIC	283	143	85	77	54	37	24	23	7*	6	6
	RDU	253	126	97	71	55	43	24	17*	9	4	7
	Median	300	163	111	79	57	48	26	17	7	5	3
	$\hat{n}(FS_xF)$	302	157	114	82	61	49	24	15	10	6	4
Summer	LGA	420	237	151	111	103	56	28*	23	5*	1	2*
	JFK	447	227	148	90	88	61	31	18	4	2	4
	EWR	405	231	150	108	88	65*	44	14	5*	5	4
	PHL	484	271	169	141	84	64	27	12	0	4	3
	BAL	468	259*	188	124	100	82	34	11	8	3*	2*
	ADW	347	211	183	118	112	74	26	12	7	1	2*
	DCA	473	259*	167*	121	96*	70	26	16	5*	4	4
	RIC	451*	265	186	129	89	59	35	17	7	6	1
	RDU	478	282	165	120*	100	88	27	15*	2	2	2*
	Median	451	259	167	120	96	65	28	15	5	3	2
	$\hat{n}(FS_xF)$	450	248	166	115	90	75	33	16	8	4	2

Table 15. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When GE Eight-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	(x) hours 1	2	3	4	5	6	12	18	24	30	36
Winter	LGA	277	180	100	96	59*	51*	15	11	11	10	5
	JFK	327	168*	116	90	74	44	17*	17	10*	8*	10
	EWR	302	177	105*	76	53	47	13	6	16	7	5
	PHL	376	226	115	95	59*	57	12	5	14	5	7*
	BAL	330	159	96	82*	72	60	16	11	8	6	6
	ADW	280	162	107	66	59*	61	19	6	10*	13	7*
	DCA	329	157	108	80	53	55	19	10*	10*	11	5
	RIC	298	146	88	84	58	48	18	9	11	9	10
	RDU	307*	168*	100	75	59*	49	30	10*	10*	7	8
	Median	307	168	105	82	59	51	17	10	10	8	7
	$\hat{n}(FS_xF)$	317	162	106	80	58	45	19	14	10	7	5
Summer	LGA	471	245	144	95	85	63	25*	15	7	5	5
	JFK	444	229	166*	101	86	80	21	12	8	7	4*
	EWR	416	255	164	111	91	58	20	12	9	2	6
	PHL	479*	284	149	140	81	56	25*	13*	5	4*	2
	BAL	524	251	166*	123	93*	65	27	18	3	4*	4*
	ADW	422	261*	181	114*	106	76	33	16	4	4*	3
	DCA	488	279	170	125	101	78	19	12	9	3	3
	RIC	539	290	172	131	96	80	24	14	6*	0	5
	RDU	521	265	151	108	103	9C	32	10	6*	3	1
	Median	479	261	166	114	93	76	25	13	6	4	4
	$\hat{n}(FS_xF)$	476	264	167	123	97	76	23	11	7	5	3

Table 16. Observed Number of Runs, $n(FS_xF)$, of x Hours in Length Observed in the Data Sample and Estimated Through the Use of Eq. (14) When Ten-tenths Sky Cover (overcast) is Considered a Success. Median values are identified with asterisks

Season	Station	x (hours) 1	2	3	4	5	6	12	18	24	30	36
Winter	LGA	329	172	119	85	68	40	14	14	6	9	5
	JFK	311	180	119	73	64	41	24	9*	10	5	4*
	EWR	325	168*	111	79	50	62	12	9*	7	8*	1
	PHL	394	185	110	83	50	42*	20	14	9*	5	6
	BAL	290	160	118	62	64	41	22	13	10	8*	4*
	ADW	291	161	123	76	60*	46	14	6	8	8*	2
	DCA	324*	189	110	80*	68	49	19	8	13	7	5
	RIC	287	164	98	85	54	52	17*	9*	11	10	6
	RDU	339	160	116*	84	51	40	21	14	5	5	4*
	Median	324	168	116	80	60	42	17	9	9	8	4
	$\hat{n}(FS_xF)$	309	168	112	78	58	46	18	13	9	6	4
Summer	LGA	425	233*	125	101	57	52	20	5*	5*	4	3
	JFK	427	231	149*	110	64	65*	20	4	6	3*	4
	EWR	414	220	113	92	69	65*	18	5*	5*	5	1
	PHL	514	302	152	109*	92	68	23	5*	5*	4	3
	BAL	416	204	123	102	85*	52	14	10	5*	1	2*
	ADW	476	254	161	123	101	69	15	8	5*	2	1
	DCA	512	264	176	109*	94	65*	15	7	9	3*	1
	RIC	463*	258	156	110	77	65*	19*	6	7	2	5
	RDU	507	233*	147	112	87	92	19*	3	3	3*	1
	Median	463	233	149	109	85	65	19	5	5	3	2
	$\hat{n}(FS_xF)$	458	239	152	109	81	63	17	9	5	3	2

Table 17. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When Zero-tenths Sky Cover (clear) is Considered a Success. Median values are identified with asterisks

		x (hours)								
Season	Station	0	1	3	5	7	9	11	13	15
Winter	LGA	.0508*	.766	.838	.889*	.863	.886	.896	.866	.845
	JFK	.0511	.773	.832	.873	.878*	.894*	.863	.860	.833
	EWR	.0502	.788	.848*	.886	.901	.884	.912	.878*	.837
	PHL	.0521	.734	.846	.856	.874	.896	.874	.822	.896
	BAL	.0507	.775*	.862	.888	.894	.929	.918	.882	.910
	ADW	.0475	.786	.862	.902	.893*	.885	.910	.878*	.879
	DCA	.0522	.763	.819	.891	.869	.891	.898*	.925	.935
	RIC	.0494	.836	.857	.913	.894	.943	.920	.928	.868
	RDU	.0512	.796	.859	.917	.897	.933	.896	.917	.874*
Summer	LGA	.0500	.705	.789	.849	.785	.859	.786	.817	.789
	JFK	.0504	.710	.793*	.796	.796	.774	.808	.818*	.800
	EWR	.0517*	.739	.811	.820	.818*	.846*	.821	.812	.870
	PHL	.0542	.690	.760	.822	.800	.800	.810*	.805	.870
	BAL	.0579	.730	.796	.842	.840	.862	.810*	.763	.872
	ADW	.0472	.769	.782	.775	.798	.854	.797	.861	.792
	DCA	.0578	.717	.770	.833	.846	.825	.828	.870	.791
	RIC	.0514	.735	.813	.830*	.833	.850	.842	.878	.823
	RDU	.0556	.724*	.808	.832	.858	.810	.789	.862	.806*

Table 18. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When LE Three-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

		x (hours)								
Season	Station	0	1	3	5	7	9	11	13	15
Winter	LGA	.0706	.772	.854	.900	.911	.916	.914*	.907	.933
	JFK	.0759	.762	.831	.890	.931	.921	.924	.919	.909
	EWR	.0733	.772	.852	.917	.914	.934	.914*	.898	.920*
	PHL	.0740	.734	.861	.895	.903	.895	.901	.913	.900
	BAL	.0726*	.741	.861	.904	.912	.939	.913	.905	.898
	ADW	.0685	.748	.853	.907	.915*	.929*	.896	.904	.932
	DCA	.0727	.742	.856*	.876	.921	.909	.894	.949	.920*
	RIC	.0701	.759*	.886	.908	.925	.935	.924	.908*	.920*
	RDU	.0682	.772	.868	.903*	.929	.934	.964	.929	.937
Summer	LGA	.0884	.737	.839	.848	.892	.900	.883	.888	.865
	JFK	.0881	.718	.838	.870	.907	.893	.890*	.850	.906
	EWR	.0910	.745	.843	.873	.902	.919	.894	.895	.903
	PHL	.0944	.719	.825	.872	.855	.876	.853	.889	.902*
	BAL	.108	.736	.820	.864*	.868	.869	.882*	.893	.935
	ADW	.0842	.774	.813	.835	.876*	.887	.919	.882*	.880
	DCA	.101	.726	.832*	.864*	.876*	.891*	.898	.848	.899
	RIC	.0937*	.733*	.808	.864*	.870	.896	.899	.861	.926
	RDU	.102	.731	.838	.864*	.888	.872	.879	.833	.844

Table 19. Relative Frequency of Success Given a Failure and x Hours of Success $RF(S|FS_x)$, Obtained From the Data Sample When GE Eight-tenths Sky Cover is Considered a Success. Median Values are identified with asterisks

Season	Station	x (hours								
		0	1	3	5	7	9	11	13	15
Winter	LGA	.0894	.781	.876	.904*	.938	.916	.957	.956	.954
	JFK	.0944	.752*	.859	.880	.936	.939	.955	.959	.968
	EWR	.0922	.763	.868	.914	.887	.934	.960	.956	.952*
	PHL	.108	.737	.861	.905	.926*	.927*	.944*	.953*	.944
	BAL	.0914*	.744	.880	.884	.928	.939	.949	.940	.958
	ADW	.0902	.776	.867*	.907	.918	.900	.940	.933	.935
	DCA	.0954	.746	.867	.915	.920	.925	.936	.946	.955
	RIC	.0866	.754	.886	.903	.906	.935	.914	.971	.938
	RDU	.0833	.749	.867	.897	.927	.914	.934	.946	.923
Summer	LGA	.0945	.705	.836	.867	.862	.898	.938	.932	.926
	JFK	.100*	.725	.824	.873	.888	.890*	.902	.934	.940
	EWR	.0969	.738	.822*	.859	.883	.886	.907	.928	.930
	PHL	.108	.724	.847	.881	.890	.904	.906*	.905	.911
	BAL	.0921	.683	.811	.843	.850	.895	.937	.920	.945
	ADW	.104	.749	.819	.849	.872*	.914	.896	.938	.938
	DCA	.100*	.716*	.821	.846	.852	.847	.887	.924*	.952
	RIC	.107	.699	.821	.854*	.879	.875	.898	.899	.934*
	RDU	.0978	.698	.839	.848	.828	.882	.930	.896	.920

Table 20. Relative Frequency of Success Given a Failure and x Hours of Success, $RF(S|FS_x)$, Obtained From the Data Sample When Ten-tenths Sky Cover (overcast) is Considered a Success. Median values are identified with asterisks

Season	Station	x (hours								
		0	1	3	5	7	9	11	13	15
Winter	LGA	.0747*	.735	.839	.873	.927	.937	.960	.940	.947*
	JFK	.0739	.745*	.837	.881	.931	.946	.943	.962	.938
	EWR	.0770	.738	.851	.910	.921	.929*	.946	.935	.933
	PHL	.0856	.700	.850	.908	.925	.920	.950	.948	.939
	BAL	.0716	.753	.838	.883	.891	.936	.940*	.956	.952
	ADW	.0759	.763	.841*	.896*	.891	.940	.929	.958	.956
	DCA	.0813	.747	.857	.882	.909	.916	.930	.958	.961
	RIC	.0724	.758	.866	.902	.917	.906	.922	.950	.951
	RDU	.0697	.718	.835	.899	.920*	.920	.938	.952*	.909
Summer	LGA	.0636	.677	.810	.868	.842	.908	.908	.910	.904
	JFK	.0704	.697	.802	.870	.847*	.920	.898*	.931	.927*
	EWR	.0675	.693	.842	.864	.858	.878*	.954	.926	.935
	PHL	.0823	.681*	.808	.827*	.882	.849	.887	.936	.894
	BAL	.0584	.668	.805	.791	.837	.880	.897	.836	.945
	ADW	.0744	.687	.796	.800	.806	.834	.903	.937	.950
	DCA	.0739	.664	.764	.796	.854	.898	.923	.914	.915
	RIC	.0697*	.682	.787	.835	.868	.850	.891	.897	.909
	RDU	.0693	.659	.803*	.821	.821	.848	.851	.916*	.932

5.2 Modeled

The probability of a run of exactly x hours in length, $P(FS_xF)$, is the probability that there will be a failure followed by x successes followed by another failure. This might be estimated as follows, for runs of length one, two, and x hours, respectively:

$$\hat{P}'(FS_1F) = \hat{P}(F)\ \hat{P}(S|F)\ \hat{P}(F|FS_1) \tag{11}$$

$$\hat{P}'(FS_2F) = \hat{P}(F)\ \hat{P}(S|F)\ \hat{P}(S|FS_1)\ \hat{P}(F|FS_2) \tag{12}$$

$$\hat{P}'(FS_xF) = \hat{P}(F)\ \hat{P}(S|F)\ \hat{P}(S|FS_1)\ \hat{P}(S|FS_2)\ldots\hat{P}(S|FS_{x-1})\ \hat{P}(F|FS_x) \tag{13}$$

where $\hat{P}(S|F)$ is the estimated probability of a success given that a failure occurred the previous hour, $\hat{P}(S|FS_1)$ is the estimated probability of a success given that a success occurred and a failure occurred two hours earlier....., $\hat{P}(F|FS_x)$ is the estimated probability of a failure given that x successes occurred the previous x consecutive hours and a failure occurred x + 1 hours earlier. The unconditional and conditional probabilities can be estimated from the relative frequencies but very large samples of data are required to obtain statistically stable relative frequencies of long runs, because they are rare events.

The points plotted in Figure 7 show the nine-station median relative frequencies of success given a failure and x hours of successes have occurred. The median values are given in Table 21. Smooth curves were subjectively drawn through the points in Figure 7. The probabilities required for the solution of Eqs. (11) to (13) were estimated from these curves.

Table 21 shows values of $\hat{P}(S|FS_x)$ that were estimated from the curves shown in Figure 7. The conditional probabilities always increase for at least 8 hours and most of the values increase for at least 11 hours.

The values found in Table 21 were used to solve Eq. (13). By substituting $\hat{P}'(FS_xF)$ from Eq. (13) for $RF(FS_xF)$ in Eq. (9) the following expression is obtained for estimating $n(FS_xF)$

$$\hat{n}(FS_xF) = \hat{P}'(FS_xF)N\ . \tag{14}$$

Solutions to Eq. (14) are given in Tables 13 to 16. The agreements between the observed number of runs and those calculated from Eq. (14) are very good. It should be understood that this is not an independent test of Eq. (14) but rather a subjective fitting to the data to obtain conditional probabilities and an objective method for finding the desired probability estimates.

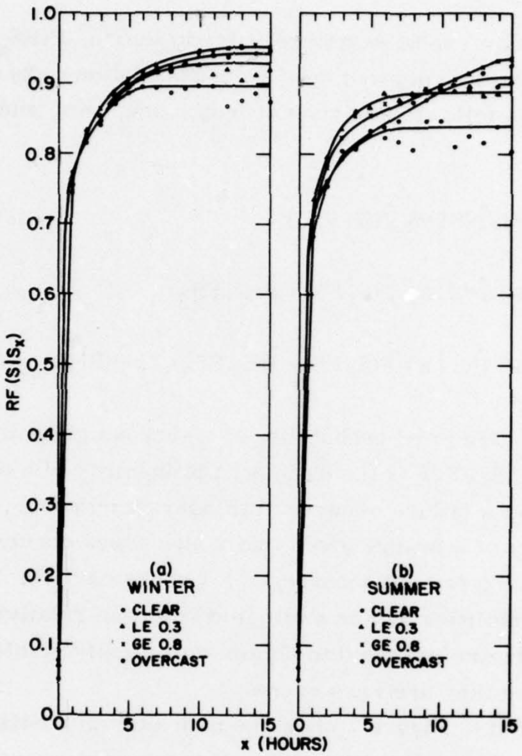


Figure 7. Relative Frequencies of Success Given Exactly x Hours of Consecutive Success Have Occurred, in Winter (a) and Summer (b). The curves were subjectively drawn

Table 21. Median Values of $RF(S|FS_x)$ Obtained From the Data Sample (Tables 17, 18, 19, and 20) and Probability Estimates $\hat{P}(S|FS_x)$ Determined From Subjectively Drawn Curves of the Medians Shown in Figure 7

Season		RF(F)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	≥15
Winter	Clear																	
	Median	.7366	.0508	.775	.826	.848	.870	.889	.894	.893	.886	.894	.896	.898	.869	.878	.887	.874
	$\hat{P}(S\|FS_x)$		.0508	.775	.820	.850	.870	.886	.893	.896	.897	.897	.897	.897	.897	.897	.897	.874
	LE 0.3																	
	Median	.6151	.0726	.759	.835	.856	.885	.903	.910	.915	.923	.929	.928	.914	.920	.908	.913	.920
	$\hat{P}(S\|FS_x)$		.0726	.759	.835	.856	.880	.898	.908	.915	.922	.925	.928	.928	.928	.928	.928	.928
	GE 0.8																	
	Median	.4978	.0914	.752	.831	.867	.883	.904	.905	.926	.923	.927	.940	.944	.952	.953	.954	.952
	$\hat{P}(S\|FS_x)$		.0914	.752	.831	.867	.885	.905	.918	.927	.936	.942	.947	.949	.950	.950	.950	.950
	Overcast																	
	Median	.5825	.0747	.745	.815	.841	.876	.896	.912	.920	.919	.929	.948	.940	.939	.952	.941	.947
	$\hat{P}(S\|FS_x)$		.0747	.747	.816	.850	.876	.895	.908	.919	.926	.932	.939	.941	.942	.942	.942	.942
Summer	Clear																	
	Median	.7922	.0517	.724	.760	.793	.808	.830	.822	.818	.809	.846	.848	.810	.825	.818	.840	.806
	$\hat{P}(S\|FS_x)$		.0517	.724	.760	.793	.808	.823	.832	.837	.839	.840	.840	.840	.840	.840	.840	.840
	LE 0.3																	
	Median	.6271	.0937	.733	.799	.832	.856	.864	.888	.876	.888	.891	.893	.890	.896	.882	.885	.902
	$\hat{P}(S\|FS_x)$		.0937	.733	.799	.832	.860	.872	.878	.882	.888	.888	.888	.888	.888	.888	.888	.888
	GE 0.8																	
	Median	.5840	.100	.716	.780	.822	.853	.854	.870	.872	.883	.890	.898	.906	.914	.924	.934	.934
	$\hat{P}(S\|FS_x)$		.100	.716	.780	.822	.840	.850	.862	.870	.880	.890	.898	.906	.914	.924	.934	.934
	Overcast																	
	Median	.7176	.0697	.681	.756	.803	.810	.827	.839	.847	.872	.878	.892	.898	.893	.916	.898	.927
	$\hat{P}(S\|FS_x)$		.0697	.681	.756	.795	.814	.830	.842	.855	.866	.878	.887	.897	.900	.906	.910	.910

6. RECURRENCE

6.1 Observed

The relative frequency of the recurrence of a success ℓ hours later given that a success occurred, $RF(S_\ell|S)$, can be determined from the data by dividing the number of occurrences of successes spaced ℓ hours apart, $n(SS_\ell)$, by the total number of successes, $n(S)$, that is,

$$RF(S_\ell|S) = \frac{n(SS_\ell)}{n(S)} . \tag{15}$$

Conditional recurrence relative frequencies based on 13 years of hourly observations taken at each of the nine stations are given, for selected hours, in Tables 22 to 25. The median values for each season are also given in the tables and plotted in Figures 8 to 11. Figure 8 shows that there is a pronounced 24-hourly period in clear weather. The period is much stronger in summer than in winter. Figures 10 and 11 show that this period is not found in the overcast of near overcast categories.

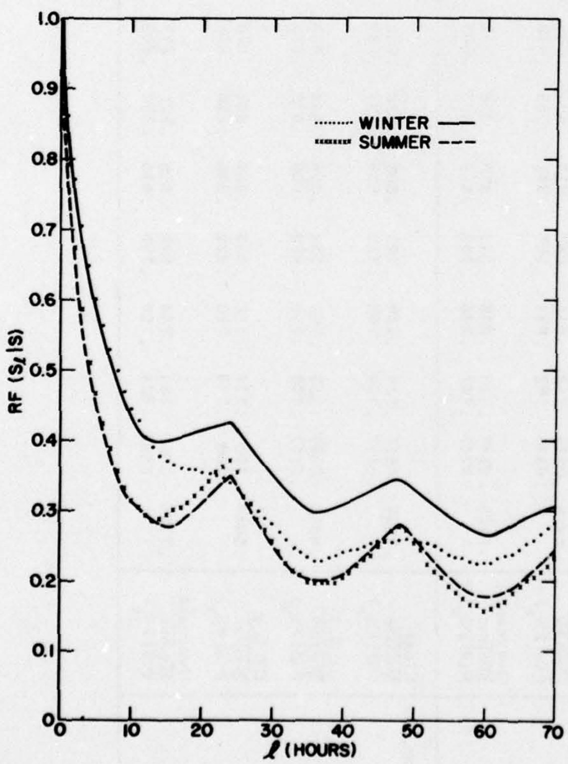


Figure 8. Relative Frequencies of a Success, Zero-tenths Sky Cover (clear), ℓ Hours Later Given a Success Has Occurred. In Winter, (dots) and In Summer (X's). The solid curve is the solution to Eq. (17) with a' = 0.225 for winter, and the dashed curve is for summer with a' = 0.265

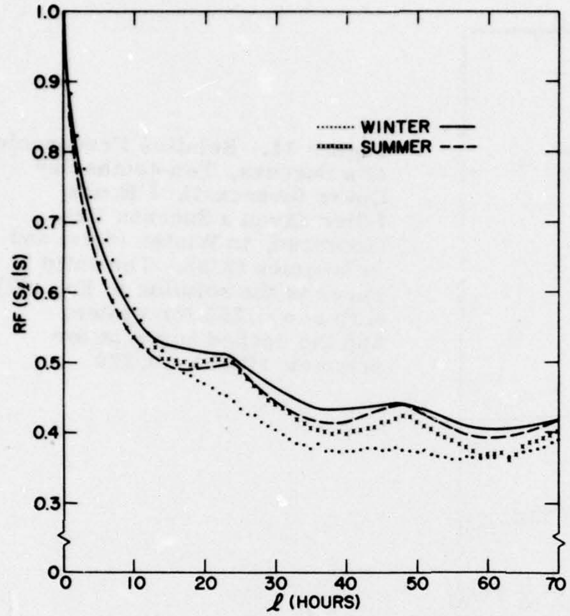


Figure 9. Relative Frequencies of a Success, LE Three-tenths Sky Cover, ℓ Hours Later Given a Success Has Occurred, in Winter (dots) and in Summer (X's). The solid curve is the solution to Eq. (17) with a' = 0.235 for winter, and the dashed curve is for summer with a' = 0.262

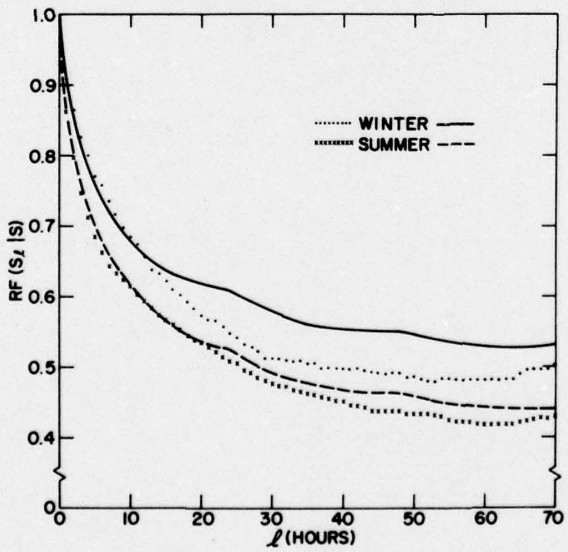


Figure 10. Relative Frequencies of a Success, GE Eight-tenths Sky Cover ℓ Hours Later Given a Success Has Occurred, in Winter (dots) and in Summer (X's). The solid curve is the solution to Eq. (17) with a' = 0.226 for winter, and the dashed curve is for summer with a' = 0.235

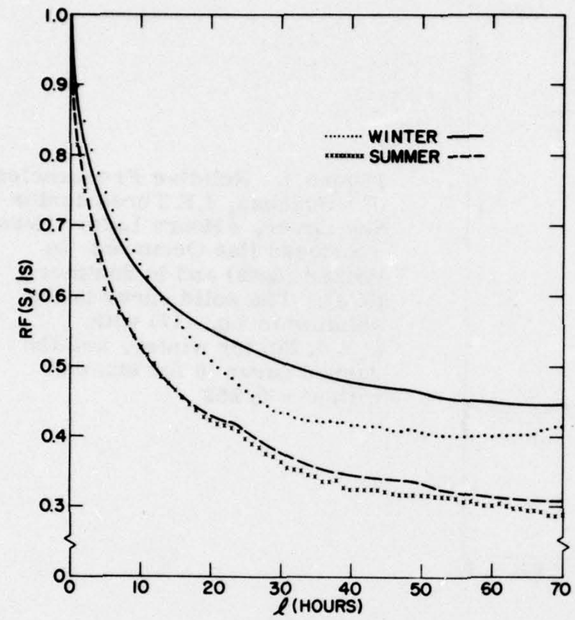


Figure 11. Relative Frequencies of a Success, Ten-tenths Sky Cover (overcast), ℓ Hours Later Given a Success Has Occurred, in Winter (dots) and in Summer (X's). The solid curve is the solution to Eq. (17) with a' = 0.232 for winter, and the dashed curve is for summer with a' = 0.234

Table 22. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success Has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When Zero-tenths Sky Cover (clear) is Considered a Success. Median values are identified with asterisks

Season	Station	P(S)	1	2	3	4	5	6	9	12	18	24	30	36	48	60	71
Winter	LGA	.2481	.846	.752	.682	.625	.577	.532	.431	.367	.320	.304	.244	.208	.247	.220	.278
	JFK	.2501	.847	.752	.681	.625	.577	.535	.438	.373	.322	.303	.250	.212	.245	.211	.266
	EWR	.2634*	.860*	.770*	.703*	.646	.599	.560	.457	.389	.340	.316	.259	.222	.258*	.226*	.287*
	PHL	.2450	.840	.748	.674	.615	.566	.525	.424	.355	.324	.323	.255	.209	.257	.202	.266
	BAL	.2737	.866	.781	.720	.667	.624	.588	.493	.425	.373	.356	.298	.254	.280	.242	.297
	ADW	.2540	.860*	.770*	.703*	.647*	.600*	.561*	.468*	.404*	.357*	.340*	.281*	.229*	.253	.218	.282
	DCA	.2716	.860*	.773	.707	.659	.618	.580	.494	.437	.385	.350	.295	.257	.269	.245	.290
	RIC	.3073	.889	.811	.752	.704	.663	.624	.538	.472	.416	.378	.323	.286	.305	.276	.321
	RDU	.3106	.886	.810	.755	.708	.672	.638	.562	.495	.432	.391	.327	.290	.314	.295	.329
	Median	.2634	.860	.770	.703	.647	.600	.561	.468	.404	.357	.340	.281	.229	.258	.226	.287
	Eq. (17)		.829	.745	.682	.629	.585	.547	.459	.404	.408	.427	.350	.298	.345	.266	.313
Summer	LGA	.1867	.782	.665	.580	.515	.458	.410	.323	.274	.279	.318	.227	.186	.244	.142	.224
	JFK	.1833	.775	.650	.564	.500	.444	.404	.325	.290	.284	.321	.221	.194	.240	.166	.205
	EWR	.2078*	.803	.683	.597	.528	.475	.436	.350	.301	.312	.350	.258*	.203	.271	.158	.243
	PHL	.1920	.772	.651	.565	.500	.450	.410	.327	.294*	.300	.355	.245	.190	.274	.158	.244*
	BAL	.2301	.806	.689	.614	.556	.508	.466	.381	.338	.357	.399	.290	.241	.312	.186	.271
	ADW	.1883	.797	.667	.577	.510	.457	.416	.329	.292	.313*	.371*	.252	.196*	.278*	.159*	.244*
	DCA	.2186	.793*	.678	.601	.546	.502	.466	.388	.349	.357	.373	.283	.241	.279	.206	.253
	RIC	.2093	.806	.686	.602	.539	.489	.450	.368	.318	.332	.391	.283	.218	.299	.198	.277
	RDU	.2101	.791	.671*	.585*	.519*	.467*	.423*	.332*	.289	.326	.400	.276	.192	.285	.156	.264
	Median	.2078	.793	.671	.585	.519	.467	.423	.332	.294	.313	.371	.258	.196	.278	.159	.244
	Eq. (17)		.751	.643	.562	.501	.451	.409	.327	.294	.292	.350	.249	.202	.280	.178	.253

Table 23. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success has Occurred, $RF(S_\ell|S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When LE Three-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	P(S)	1	2	3	4	5	6	9	12	18	24	30	36	48	60	71
Winter	LGA	.3739	.882	.812	.759	.717	.683	.650	.575	.520	.451	.404	.370	.350	.361	.353	.379
	JFK	.3821	.877	.811	.761	.721	.686	.654	.578	.528	.459	.418	.374	.355	.371	.361	.386
	EWR	.3895	.885*	.819	.768	.730	.699	.668	.595	.540	.475	.427	.389	.367	.375*	.363	.394
	PHL	.3636	.870	.804	.753	.711	.676	.646	.569	.514	.453	.417	.368	.345	.364	.340	.372
	BAL	.3911	.887	.827	.783	.746	.712	.684	.614	.560	.487	.446	.408	.387	.384	.372	.397
	ADW	.3809	.889	.827	.777	.737*	.705*	.676*	.598*	.543*	.480*	.447	.405*	.374*	.378	.356	.392*
	DCA	.3849*	.884	.823*	.776*	.739	.706	.677	.607	.554	.486	.445*	.408	.382	.373	.362*	.387
	RIC	.4045	.897	.839	.794	.756	.723	.697	.628	.574	.509	.462	.422	.396	.393	.390	.416
	RDU	.4194	.906	.852	.810	.777	.751	.726	.665	.615	.541	.491	.443	.414	.413	.409	.429
	Median	.3849	.885	.823	.776	.737	.705	.676	.598	.543	.480	.445	.405	.374	.375	.362	.392
	Eq. (17)		.862	.797	.750	.710	.677	.650	.584	.543	.519	.509	.465	.436	.442	.405	.418
Summer	LGA	.3703	.850	.772*	.718*	.676*	.641*	.615*	.550*	.512	.472	.468	.421	.387	.402	.345	.377
	JFK	.3719	.851*	.777	.725	.684	.647	.617	.549	.509	.477	.470	.422	.397	.406	.357	.374
	EWR	.3920	.859	.782	.730	.689	.656	.629	.566	.525	.499*	.485	.440*	.407	.423*	.365*	.403
	PHL	.3645	.835	.753	.699	.658	.628	.602	.540	.507	.485	.474	.415	.383	.402	.351	.380
	BAL	.4270	.855	.786	.738	.704	.678	.655	.602	.572	.547	.532	.485	.453	.464	.417	.442
	ADW	.3646	.853	.768	.708	.665	.634	.609	.549	.517*	.505	.500*	.436	.402*	.420	.351	.388
	DCA	.4066	.852	.777	.726	.690	.662	.639	.591	.568	.542	.525	.472	.451	.446	.404	.412
	RIC	.3729*	.842	.758	.702	.658	.628	.603	.544	.510	.497	.511	.447	.401	.424	.368	.400*
	RDU	.3923	.842	.763	.707	.664	.630	.607	.552	.518	.505	.511	.456	.408	.425	.376	.412
	Median	.3729	.851	.772	.718	.676	.641	.615	.550	.517	.499	.500	.440	.402	.423	.365	.400
	Eq. (17)		.855	.777	.719	.675	.640	.611	.549	.517	.491	.500	.446	.418	.441	.393	.420

Table 24. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success Has Occurred, $RF(S_\ell | S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When GE Eight-tenths Sky Cover is Considered a Success. Median values are identified with asterisks

Season	Station	P(S)	1	2	3	4	5	6	9	12	18	24	30	36	48	60	71
Winter	LGA	.4962	.909*	.859	.822	.793	.768	.747	.688	.642	.572	.521	.490	.478	.481	.484	.491
	JFK	.5022*	.906	.858	.822	.794	.769	.746	.687	.640	.572	.527	.495	.479	.488	.488*	.498
	EWR	.5073	.910	.862*	.825	.796	.773	.751	.694	.650	.585	.532	.500	.486	.492	.490	.506*
	PHL	.5306	.904	.858	.827	.801*	.780	.760	.708	.670	.605	.566	.535	.523	.525	.515	.530
	BAL	.4988	.908	.862*	.829*	.799	.774	.753*	.701	.657*	.589	.544	.515*	.502*	.490*	.477	.492
	ADW	.5068	.912	.865	.831	.802	.775*	.751	.698	.658	.594	.556	.525	.510	.499	.484	.511
	DCA	.5165	.910	.868	.836	.808	.783	.759	.707	.666	.604	.561	.535	.521	.506	.496	.512
	RIC	.5006	.914	.868	.835	.806	.781	.758	.704	.660	.601	.554	.520	.502*	.490*	.488*	.510
	RDU	.4761	.908	.863	.829*	.801*	.777	.754	.699*	.653	.591*	.549*	.510	.484	.475	.473	.482
	Median	.5022	.909	.862	.829	.801	.775	.753	.699	.657	.591	.549	.515	.502	.490	.488	.506
	Eq. (17)		.897	.847	.811	.782	.758	.737	.689	.657	.625	.608	.579	.560	.551	.530	.530
Summer	LGA	.4118	.865	.805	.764	.731	.702	.680	.626	.591	.531	.502	.465	.444	.423	.404	.408
	JFK	.4376	.871	.811	.767	.735	.706	.686	.637	.601	.549	.524	.490	.477	.455	.433	.433
	EWR	.4282	.870	.806	.765	.732	.706	.684	.632	.598	.544*	.508*	.477*	.457	.438*	.417*	.426*
	PHL	.4387	.862*	.799	.759	.725	.703	.680	.627	.597	.558	.533	.492	.473	.459	.436	.440
	BAL	.3738	.846	.781	.738	.703	.674	.652	.601	.565	.512	.481	.445	.425	.408	.383	.371
	ADW	.4344	.865	.797*	.748*	.712*	.683	.662*	.625*	.597	.554	.537	.497	.484	.472	.442	.440
	DCA	.4033	.852	.782	.735	.703	.677	.655	.616	.588	.532	.508*	.468	.460*	.437	.417*	.406
	RIC	.4160*	.850	.784	.743	.709	.684*	.661	.617	.592*	.552	.523	.484	.464	.441	.430	.427
	RDU	.3858	.844	.774	.725	.687	.656	.631	.585	.557	.524	.502	.459	.431	.414	.397	.400
	Median	.4160	.862	.797	.748	.712	.684	.662	.625	.592	.544	.508	.477	.460	.438	.417	.426
	Eq. (17)		.866	.804	.760	.728	.702	.680	.625	.592	.548	.528	.495	.477	.464	.444	.441

Table 25. Relative Frequency of the Recurrence of a Success ℓ Hours After a Success has Occurred, $RF(S_\ell | S)$, Observed in the Data Sample and Estimated Through the Use of Eq. (17) When Ten-tenths Sky Cover (overcast) is Considered a Success. Median values are identified with asterisks

Season	Station	P(S)	1	2	3	4	5	6	9	12	18	24	30	36	48	60	71
Winter	LGA	.4091	.892	.838	.800	.768	.741	.716	.652	.590	.504	.440	.403	.390	.399	.395	.404
	JFK	.4114	.894	.841	.803	.773*	.745*	.717	.650	.593	.503	.439	.402	.387	.395	.395	.401
	EWR	.4262	.896*	.845*	.805*	.772	.744	.718	.657	.602	.516	.453	.416	.407	.410	.407	.419*
	PHL	.4528	.897	.850	.815	.786	.760	.738	.677	.628	.547	.491	.461	.452	.449	.438	.451
	BAL	.4155	.899	.847	.808	.776	.749	.726	.662*	.610*	.529*	.472*	.434*	.423*	.411*	.388	.402
	ADW	.4246	.897	.844	.805*	.772	.744	.720*	.662*	.616	.537	.481	.452	.436	.418	.402*	.422
	DCA	.4395	.896*	.846	.812	.780	.751	.727	.671	.624	.548	.493	.462	.449	.428	.415	.430
	RIC	.4175*	.899	.849	.810	.779	.751	.727	.666	.620	.542	.480	.440	.426	.411*	.403	.421
	RDU	.3855	.889	.836	.796	.766	.738	.714	.649	.596	.522	.471	.428	.401	.388	.379	.387
	Median	.4175	.896	.845	.805	.773	.745	.720	.662	.610	.529	.472	.434	.423	.411	.402	.419
	Eq. (17)		.877	.822	.781	.748	.722	.699	.647	.610	.561	.533	.505	.487	.468	.451	.445
Summer	LGA	.2801	.836	.764	.717	.680	.650	.619	.554	.502*	.432	.385	.337	.309	.294	.274	.276
	JFK	.3015	.837	.767	.719	.681	.650	.621	.564	.520	.452	.408	.368	.344	.325	.302	.299
	EWR	.3050	.846	.773	.726	.688	.655	.629	.566	.524	.451	.407*	.364*	.338*	.317*	.301*	.300
	PHL	.3188	.824*	.753	.708	.669	.638	.612	.548	.515	.462	.429	.378	.356	.343	.321	.328
	BAL	.2526	.825	.749*	.692*	.646	.611	.583	.524	.487	.414	.385	.335	.312	.295	.264	.257
	ADW	.2889	.817	.732	.674	.632	.600	.576	.528	.495	.437*	.409	.367	.353	.324	.302	.296
	DCA	.2824*	.812	.732	.683	.649*	.618*	.596*	.542*	.505	.437*	.401	.367	.344	.322	.305	.287*
	RIC	.2736	.815	.734	.685	.644	.616	.587	.530	.488	.441	.411	.361	.337	.314	.293	.283
	RDU	.2534	.796	.705	.642	.595	.557	.525	.480	.450	.400	.384	.333	.296	.282	.259	.269
	Median	.2824	.824	.749	.692	.649	.618	.596	.542	.502	.437	.407	.364	.338	.317	.301	.287
	Eq. (17)		.835	.760	.707	.665	.632	.605	.544	.502	.443	.418	.377	.357	.338	.315	.309

6.2 Modeled

McAllister[5] proposed an expression of the form

$$\hat{P}(S_{t+\ell}|S_t) = P(S_{t+\ell}) + [1-P(S_t)]\ e^{-a\ell^b} \tag{16}$$

for estimating recurrence probabilities of cloud cover. He used a = 0.263 and b = 0.632 as the best estimates of the parameters. Gringorten[6] showed that Eq. (16) yields probability estimates very close to those obtained from the bivariate normal distribution if the parameter b is fixed at 0.620 and a is allowed to vary with the climatic frequency of the event and the basis persistence of the element.

Eq. (16) was modified to; (1) eliminate any possibility of obtaining probability estimates greater than one; (2) take into account diurnal periods in weather events and (3) obtain climatic estimates independent of the initial hour. The new equation was expressed as follows

$$\hat{P}(S_\ell|S) = \frac{1}{P(S)}\left[(1-e^{-a'\ell^b})\,(\overline{YZ}) + e^{-a'\ell^b}\ (\overline{W})\right] \tag{17}$$

where $\overline{YZ}$ is the temporal average of the product of the two probabilities, that is,

$$\overline{YZ} = \frac{1}{24}\sum_{t=0}^{23}(Y_t Z_{t+\ell}) \tag{17a}$$

where Y and Z are probabilities of success at time t and $t+\ell$ hours, respectively; and, $\overline{W}$ is the temporal average of the lower of each pair of probabilities, that is,

$$W = \frac{1}{24}\sum_{i=0}^{23} W_i \tag{17b}$$

where $W_i = Y_t$ or $Z_{t+\ell}$ whichever is smaller.

Table 26 shows that there is a pronounced diurnal period in sky cover occurrences. The hourly climatic frequencies of the events were substituted into Eq. (17) using ℓ = 12 hours and b = 0.620 and the equation was solved to find the parameter a'. The a' values are given in Table 27.

---

5. McAllister, C.R. (1969) Cloud-cover recurrence and diurnal variations. J. Appl. Meteor. 8:769-777.
6. Gringorten, I.I. (1971) Modeling conditional probability, J. Appl. Meteor. 10:646-657.

Table 26. Nine-station Median Relative Frequency of Each Sky Cover Category for Each Hour of the Day

Hour (LST)	Winter Category 0.0	LE 0.3	GE 0.8	1.0	Summer Category 0.0	LE 0.3	GE 0.8	1.0
00	.351	.458	.463	.399	.339	.492	.362	.271
01	.361	.450	.469	.422	.362	.492	.379	.272
02	.352	.444	.470	.420	.344	.503	.377	.293
03	.349	.439	.480	.422	.350	.489	.376	.296
04	.344	.438	.480	.426	.278	.454	.391	.300
05	.338	.439	.490	.432	.168	.372	.451	.315
06	.293	.407	.498	.427	.182	.352	.483	.334
07	.204	.338	.543	.439	.192	.367	.474	.342
08	.193	.319	.573	.458	.204	.379	.451	.315
09	.191	.326	.567	.445	.202	.384	.428	.291
10	.186	.335	.550	.435	.175	.375	.420	.271
11	.177	.317	.543	.423	.126	.329	.431	.273
12	.170	.315	.531	.421	.093	.285	.443	.265
13	.164	.321	.543	.422	.076	.256	.445	.259
14	.168	.309	.537	.426	.070	.252	.444	.247
15	.171	.326	.532	.420	.079	.259	.426	.237
16	.181	.337	.531	.408	.095	.289	.410	.249
17	.197	.359	.503	.392	.117	.329	.421	.257
18	.245	.403	.478	.400	.153	.352	.423	.275
19	.307	.429	.472	.397	.149	.350	.444	.280
20	.332	.438	.467	.405	.206	.387	.410	.288
21	.338	.441	.462	.407	.260	.443	.384	.283
22	.343	.448	.466	.413	.316	.469	.368	.278
23	.350	.447	.472	.417	.325	.487	.366	.279

Table 27. The "a'" Values Used in Eq. (17) to Find the Curves Shown in Figures 8 to 11

Season	Clear	LE 0.3	GE 0.8	Overcast
Winter	0.225	0.235	0.226	0.232
Summer	0.265	0.262	0.235	0.234

Eq. (17) was solved for lags from 1 to 71 hours using the a' values given in Table 27. The resulting curves are shown in Figures 8 to 11. The fits to the summer relative frequencies are excellent for all sky cover categories. The fits to the winter values are poor after 12 hours.

7. REMARKS

Relative frequencies of persistence, runs and recurrence of sky cover along the east coast of the United States between New York and North Carolina, presented in this report, are based on more than 250,000 hourly observations taken in winter and a similar number taken in summer. They are believed to be good approximations of the true probabilities.

Models are presented for use in estimating joint and conditional probabilities. The estimates are usually in good agreement with the relative frequencies when the parameters are carefully chosen. However, the best parameters for the Central East Coast area of the United States may not be the best for other geographical areas. Future studies will be extended to other areas and to improving the models. Other weather elements are under study at the present time.

References

1. Lund, I. A., and Grantham, D. D. (1977) Persistence, runs and recurrence of precipitation, J. Appl. Meteor. 16:346-358.
2. U. S. Department of Commerce (1975) Federal Meteorological Handbook No. 1, Surface Observations, U. S. Government Printing Office, Washington, D. C. 309 pp.
3. Gringorten, I. I. (1966) A Stochastic model of the frequency and duration of weather events, J. Appl. Meteor. 5:606-624.
4. Keilson, J., and Ross, H. F. (1975) Passage time distributions for Gaussian Markov (Ornstein-Uhlenbeck) statistical processes, Selected Tables in Mathematical Statistics Vol. III. American Mathematical Society, Providence, Rhode Island, pp 233-327.
5. McAllister, C. R. (1969) Cloud-cover recurrence and diurnal variation, J. Appl. Meteor. 8:769-777.
6. Gringorten, I. I. (1971) Modeling conditional probability, J. Appl. Meteor. 10:646-657.